MATHEMATICS 136.275
EXAM
3 hours.
April 14, 2004

Instructions: Attempt all questions. The total value of all questions is 100. Values of individual questions are printed beside the statement of the question. If you need more space use the reverse side of the page, but indicate clearly that you are doing so. There are two blank pages at the end of the test for you to use as scrap paper.
Please fill in the information requested below. Good luck!

Name ____

Signature _____________________________________

Student No. ___________________________________

QUESTION#	MARK
1	/8
2	/12
3	/9
4	/12
5	/10
6	/11
7	/8
8	/10
9	/9
10	/11
TOTAL	/100

Value

[2] 1.a) Write the definition of the limit of a sequence.

b) Prove **by using the definition** of the limit that

$$\lim_{n\to\infty}\left(\frac{3n-2}{n}+\frac{1}{n}\cos(n\pi)\right)=3$$

[6]

2. Find if the following series converge or not. Show all of your work and **state which test** are you using.

a) $\displaystyle\sum_{n=1}^{\infty}\frac{5n}{3(n^2+2n)}$

[3]

b) $\displaystyle\sum_{n=1}^{\infty}\left(\frac{1}{3}-e^{-n}\right)^n$

[3]

c) $\displaystyle\sum_{n=2}^{\infty}\frac{\left(2+(-1)^n\right)\ln n}{n!}$

[4]

3. a)Find the radius of convergence of the power series $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{(x-3)^n}{2^n n}$ and show that the series converges conditionally at one end of the interval of convergence, and that it diverges at the other.

[5]

b) If $f(x)=\sum_{n=1}^{\infty}(-1)^{n+1}\frac{(x-3)^n}{2^n n}$ (over the interior of the interval of convergence of the series) find $f^{(4)}(3)$ and find the series representation of $f'(x)$.

[4]

c) Can $\lim_{N\to\infty} R_N(4)=1$, where $R_N(4)$ is the N-th reminder for f in b).

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4. a) Write a formula for the curvature k(t) of the $\mathbb{R}^3$ curve **r**(t).
[2]

b) Let **r**(t) =(t , 1, 3-t^2). Find a point on **r**(t) with maximal curvature.
[5]

c) Show that the curve in b) is on the surfaces $z = 4 - x^2 - y^2$ and y=1.
Draw both surfaces and guess what kind of a curve **r**(t) is.
[3]

5. Let $f(x,y) = \begin{cases} 0 & , xy \neq 0 \\ 1 & , xy = 0 \end{cases}$.

a) Show that f(x,y) is not continuous at (0,0) .
[2]

b) Show that $f_x(0,0)=f_y(0,0)=0$ by using the limit definition.
[5]

c) Show that f is not differentiable at (0,0) by using the definition of the derivative. (You have f_x and f_y at (0,0) from b).)

[4]

d) Show that f is not differentiable at (0,0) by stating a theorem and using part a).

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6. Let $f(x,y) = 4-x^2-y^2$ and let R be the region inside $x^2+y^2 = 4$, below y=1 and above y=–1.

a)Draw a picture of the portion of the surface determined by f that is over the region R.

[2]

b) Find all local and absolute extremes of f over the region R. Use the Second Partial Derivatives test.

[6]

7. Let z=f(s,t) be differentiable everywhere and let $\frac{\partial f}{\partial s}(0,2) = -1$

and $\frac{\partial f}{\partial t}(0,2) = 3$. Let s= ln(x^2+ y^2) and t= $2e^{-\sin(\pi(x+y))}$.

a)Find the direction of the greatest rate of change of z with respect to x and y when x=0 and y=1. (Hint: Find s(0,1), t(0,1) and use chain rule.)

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b)Write an equation of the tangent plane to the graph of z at x=0 and y=1 given that than z=2. (Hint: Take z=g(x,y) and g(0,1)=2)

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8. Let G be the ellipsoidal region $4x^2 + 9y^2 + \frac{z^2}{4} \le 25$. Let T^{-1} from (x,y,z) plane to the (u,v,w) plane be given by u=ax, v=by and w=cy . Choose a, b and c such that G is mapped into a spherical region in the (u,v,w) plane and use change of variables to calculate:

$$\iiint_G x^2 y \, dV \, .$$

[6]

9. Let G be the solid inside $z=4 - x^2 - y^2$, below $z=2$ and above $z=0$.

a) Calculate the volume of G.
[4]

b) Find the surface area of the part of $z=4 - x^2 - y^2$ that is on G.
[5]

10. Let **F** (x,y) = $(x^2 + y^2, -x)$ and let C be the curve given by $x^2 + y^2 = 1$.

a) Calculate the work done by **F** on a particle that moves along C in a counterclockwise direction **using Green's theorem**.

[5]

b) Calculate directly the work done by **F** on a particle that moves along C in a counterclockwise direction, i.e. by **using curve integrals**.

[4]

c) Is **F** conservative over the closed unit disc ? Explain.

[2]