136.275, Assignment No. 6

March 16, 2005

The assignment is due Wednesday, March 23 in class. Late assignments receive a mark zero.

1. Let f(x,y)= $\begin{cases} \frac{1}{2}, & (x,y) \neq (1,2) \\ 0, & (x,y) = (1,2) \end{cases}$ and let R = [0,2]X[0,4].

   a) Draw a picture of the graph of f over the rectangle R. [1]

   b)For ε>0, let P_ε be the partition of R such that on [0,2]: x_0=0, x_1=1−$\frac{\varepsilon}{2}$, x_2=1+$\frac{\varepsilon}{2}$ and x_3=2, while on [0,3]: y_0=0, y_1=2− $\frac{\varepsilon}{2}$, y_2=2+$\frac{\varepsilon}{2}$ and y_3=4.
   Draw a picture with the partition P_ε of R.
   Write and calculate the double Riemann sum for f with partition P_ε and with the choice of points: middle of each of the subrectangles . [6]

   c) If f is integrable (which it is) guess the value of $\iint_R f(x,y)dA$ by using part b). Can you find $\iint_R f(x,y)dA$ by using Fubini's theorem. Explain. [3]

2. a) In evaluating a double integral over a region G, a sum of two integrals was obtained as follows:

$$\iint_G f(x,y)dA = \int_0^1 \int_0^{2y} f(x,y)dxdy + \int_1^3 \int_0^{3-y} f(x,y)dxdy.$$

   Sketch the region G and express the double integral as an iterated integral with reversed order of integration. [3]

   b) Calculate the integral for f(x,y) = $\sin\frac{\pi x}{2}$ by using a). [6]

3. Find the surface area of the portion of the cylinder $y^2 + z^2 = 9$ that is above the rectangle R = [0,2] X [-3,3] . [6]

4. Rewrite the integral $\int_0^4 \int_4^{8-y} \int_0^{\sqrt{4-y}} f(x,y,z)dzdxdy$ as integral in dxdydz and dydxdz.
   Draw a sketch of the solid over which you are integrating and of the regions in the yz and xz planes. [9]

5. Question No. 38 on page 1075 of the textbook. [9]

Total [43/42]